Real-time Kd-tree Based Importance Sampling of Environment Maps

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Environment maps

- Environment maps are commonly used for modeling natural lighting to create realistic images.
- Complex real-world illumination can be represented efficiently by environment maps.
- High quality rendering of scenes under image-based lighting requires efficient sampling strategies.
Sampling strategies

- Environment map sampling
- Bidirectional Reflectance Distribution Function (BRDF) sampling
- Product sampling
- Multiple Importance Sampling (MIS) [33]
Motivation

- Most of the environment map sampling methods can be used in Monte Carlo simulations but they are not suitable for real-time rendering.
- We introduced a new method based on Kd-tree structure that can be used in real-time Monte-Carlo simulations.
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Related Work

Environment map sampling
- Stratified sampling. [2]
- Hierarchical sampling. [7]
- Structural importance sampling. [1]
- Blue noise sampling. [25]
- Interleaved sampling. [17]
- Inversion of the CDF. [31, 19]

Product Sampling
- Bidirectional importance sampling. [3]
- Using wavelet transforms. [5, 12]
- Using spherical harmonics. [13]

Direct illumination from environment maps
- Prefiltered environment maps. [14, 15]
- Irradiance environment maps. [27]
- Frequency space environment map rendering. [28]
- Precomputed radiance transfer [32]
- Non-linear wavelet lighting [21]
- Real-time filtered importance sampling [18]
Inversion method

- Inversion method is based on the observation that cumulative distribution functions (CDFs) range uniformly over the interval \((0, 1)\).
- If \(u\) is a uniform random number on \((0, 1)\), then using \(X = F^{-1}(u)\) generates a random number \(X\) from a distribution with specified CDF \(F\).
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Overview 1

- An environment map defined as a $w \times h$ rectangular block can be treated as a bivariate probability distribution after normalization.
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- This rectangular block is divided into sub-blocks using a Kd-tree structure.

- Empirical probabilities corresponding to each sub-block are obtained as sum of the normalized pixel intensities within each sub-block.
Overview 1

- An environment map defined as a $w \times h$ rectangular block can be treated as a bivariate probability distribution after normalization.
- This rectangular block is divided into sub-blocks using a Kd-tree structure.
- Empirical probabilities corresponding to each sub-block are obtained as sum of the normalized pixel intensities within each sub-block.
- These empirical probabilities are then sorted in descending order and corresponding block indices are assigned in increasing order.
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- This rectangular block is divided into sub-blocks using a Kd-tree structure.

- Empirical probabilities corresponding to each sub-block are obtained as sum of the normalized pixel intensities within each sub-block.

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The plots of these probabilities against sub-block indices can be considered as empirical distribution of these sub-block indices.
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The resulting estimated model can be used to generate samples for incoming light.
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Kd-tree Construction

- Keep a list of current sub-blocks.
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Choosing a sub-block for splitting

- Choosing the most convenient sub-block for splitting needs a special handling.
- We proceed to choose the sub-block having the largest intensity variation first.
- Various measures of variation can be used for this purpose.
- We tested for range, variance and sum of squared error measures respectively.
Figure 1: Rendered environment maps (Uffizi) (a): 2048 × 1024 resolution environment map requiring 8MBs of memory. (b), (c), (d): Same environment map compressed to 48KB (1:170 compression) using range, variance, and SSE criteria, respectively. Environment maps used in this work are a courtesy of Debevec.
In this work, we propose to use SSE which is defined by

\[ SSE = \sum_{i=1}^{w_b} \sum_{j=1}^{h_b} (f_{ij} - \bar{f})^2, \]  

(1)

as a selection criterion where \( \bar{f} \) is the sub block mean, \( w_b \) and \( h_b \) are the sub-block dimensions.
Splitting a selected sub-block

The splitting plane position is determined in such a way that the pooled variance [16] of the children blocks is minimum. It can be shown that minimization of the pooled variance can be reduced to maximizing the sum of squares of sub-block totals divided by their respective number of pixels.
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Empirical block probabilities are sorted in descending order. Therefore, the corresponding pdf is expected to be an exponential type distribution. We approximate this pdf by the following monotonically decreasing function

\[ p(x) = \frac{1}{\log \left( \frac{1 + \frac{n}{\alpha}}{\alpha + x} \right)}, \quad 0 \leq x \leq n, \quad (2) \]

where \( n \) is the total number of sub-blocks in the kd-tree, and \( \alpha \) is the parameter of the distribution.
Figure 2: Empirical pdfs of various environment maps and fitted analytical pdfs for uffizi(left) and doge2(right) environment maps.
Summary of preprocessing steps

- Construct the Kd-tree.
- Average sub-block intensities in the kd-tree are sorted in descending order to obtain the empirical pdf of the block indices.
- The empirical pdf is then approximated by an analytical model.
- The parameter of the pdf and the sub-block bounds are stored for use in the sampling procedure.
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Our importance sampling strategy simply consists of generating sub-block indices first and then generating incoming light direction within this block.

Sub-block indices can be generated using the well-known probability integral transformation method of obtaining random samples from a known distribution.

The following inverse function of the cdf is used for generating sub-block indices:

\[ x = P^{-1}(\xi) = \alpha \left( \left(1 + \frac{n}{\alpha}\right)^{\xi} - 1 \right), \]  

(3)

where \( \xi \) is a uniform (0,1) random variable.
We know that the approximated PDF is uniform within the selected sub-block.

Light direction can easily be generated by uniformly sampling the elevation and azimuth angles.

Within the bounds of the sub-block, we generate two uniform random variables corresponding to elevation and azimuth angles to obtain a random incoming light direction.
Summary of environment map sampling

- Generate three random variables: $\xi_1, \xi_2, \xi_3$.
- Select the corresponding block index $x = P^{-1}(\xi_1)$.
- Read the bounds of the selected sub-block.
- Generate elevation and azimuth angles uniformly within the bounds of the selected sub-block using $\xi_2, \xi_3$.
- The probability of this sample can be computed with $p(x)/\text{Area(sub-block)}$. 

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Real-time rendering implementations of our method and inversion method were made using OpenSceneGraph [4], NVIDIA CUDA [24], and Random123 [30] libraries.

Our method, and the inversion method were also implemented using Physically Based Rendering Toolkit (PBRT) [26] for off-line renderings.

All programs were executed on an Intel Core i7-920 (2.67 GHz) with 12GBs of RAM and NVIDIA GeForce GTX 480 GPU.
Figure 3: Comparison of inversion method (left) and our method (right) in real-time rendering. In this scene, the chrome-steel teapot has been rendered with both methods using 16 samples/pixel for testing real-time rendering performance.
**Figure 4:** Comparison of our method and inversion method in real-time rendering. Both of the methods were rendered with 16 samples/pixel. The FPS rates are measured under different environment maps.
Figure 5: FPS rates of our method and the inversion method for different sample sizes and environment maps.
Figure 6: Rendered spheres based on different materials and different sampling methods. Rows show different materials and columns show reference images, inversion method, and kd-tree method respectively. Insets show the scaled difference between the methods and reference images.
Figure 7: Rendered spheres using anisotropic Ward BRDF model with parameters $\alpha_x = 0.5, \alpha_y = 0.001$. (a) Reference image, (b) our kd-tree based importance sampling method (c) Křivánek and Colbert's real-time filtered importance sampling method.
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Our kd-tree based importance sampling method:

- is more suitable than inversion method for real-time rendering;
- can handle every type of material including anisotropic materials;
- can be extended to multi-dimensional functions such as BRDFs.


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Importance sampling spherical harmonics.

Approximation of glossy reflection with prefiltered environment maps.

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An alternative to null-hypothesis significance tests.  

Efficient illumination by high dynamic range images.  

Real-time shading with filtered importance sampling.  

Adaptive numerical cumulative distribution functions for efficient importance sampling.  

Probability trees.  

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All-frequency shadows using non-linear wavelet lighting approximation.  

[22] Addy Ngan, Frédéric Durand, and Wojciech Matusik.  
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Fast primitive distribution for illustration.

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Measuring and modeling anisotropic reflection.

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